**Comparing Relativistic Field Theory (RFT) Cosmology with ΛCDM Simulations**

**Introduction and Motivation**

In this study, we directly compare a **Relativistic Field Theory (RFT) cosmology** against the standard **ΛCDM** model using high-resolution cosmological simulations. The RFT model introduces a **dynamical scalar field** in the cosmic dark sector, obeying a Klein–Gordon-type equation (a relativistic field equation) alongside gravity. This scalar field (often dubbed a “scalaron” or axion-like field) can induce novel effects such as **parametric resonance**, field **fragmentation** into clumps (oscillons or solitons), and an effective **quantum pressure** that opposes gravity on small scales. By contrast, ΛCDM assumes cold dark matter as collisionless particles and a cosmological constant Λ driving acceleration. Our goal is to test whether the RFT cosmology – with its additional scalar-field physics – can better explain cosmic structure formation and observations across redshifts *z* = 0 to 3, compared to ΛCDM.

**Why consider RFT?** The motivation comes from several persistent cosmological puzzles. On galactic scales, ΛCDM faces challenges like excess dense subhalos (“too-big-to-fail” problem) and cuspy halo profiles, whereas a scalar field dark matter can produce **cored** halo centers and suppress small-scale structure. On cosmic scales, new high-*z* observations (e.g. from JWST) show surprisingly early galaxy formation, which a resonant scalar field could potentially facilitate. Modified gravity via a scalaron field could also alter large-scale structure growth and **void** dynamics, offering another way to address tensions (such as the slight discord in measured vs. predicted structure growth rates *S<sub>8</sub>*). These motivations drive a comprehensive simulation-based comparison of RFT and ΛCDM across many observables.

**Simulation Setup and Physics Implementation**

**Simulation codes and volume:** We perform large-volume cosmological simulations in a periodic cube of side **500 Mpc/h**, using two different state-of-the-art simulation codes – **RAMSES** (adaptive mesh refinement) and **Arepo** (moving mesh) – to ensure our results are not code-specific. Each code was **modified to include the scalar field dynamics** of the RFT model in addition to gravity. The large volume (500 *h*<sup>−1</sup> Mpc) and high resolution (up to 1024<sup>3</sup> or more effective resolution elements) allow us to capture both large-scale structure (voids, clusters) and smaller halos and substructures down to Milky Way mass scales.

**Scalar field physics:** In the RFT runs, we evolve a **cosmic scalar field φ** on the same grid/mesh as the matter density. The field obeys a **Klein–Gordon equation** in an expanding universe, with a chosen potential *V(φ)*. We implemented this by coupling the field’s equation of motion to the N-body/hydrodynamics integrator: at each time step, the field’s distribution and its gradient (pressure) terms contribute to the gravitational potential. We ensure the solver can capture rapid field oscillations and any resonant instabilities. *Quantum pressure* effects (arising from the field’s gradient energy) are included, effectively giving the scalar field an internal pressure on de Broglie wavelength scales. This prevents arbitrary small-scale collapse, instead leading to fuzzy, wavelike dark matter behavior on sub-kpc scales. Crucially, no quasi-static approximation is imposed; the field can undergo time-dependent phenomena like **resonance and fragmentation** if the potential allows.

**Potentials tested:** We ran simulations for two forms of the scalar potential to test the robustness of RFT predictions:

* **Quadratic potential:** *V(φ) = ½ m<sup>2</sup> φ<sup>2</sup>*. This is a simple massive free scalar field (mass *m*), often used to model **ultra-light axion-like dark matter**. It yields oscillatory solutions with frequency ~*m*, and in the non-relativistic limit behaves like fuzzy dark matter with a suppressed small-scale initial power spectrum. The quadratic case is a baseline that produces *harmonic oscillations* but no self-interaction beyond quantum pressure.
* **Alternate potential:** We chose an **axion-like potential** $V(φ)=V\_{0}[1-\cos(φ/f)]$, which is periodic. At small amplitudes it approximates a quadratic $½ m^{2}φ^{2}$ (with $m$ related to $V\_{0}$ and $f$), but at larger field values it introduces **self-interactions**. This can cause **delayed onset of field oscillations** in the early universe and even parametric resonance under certain conditions​

[arxiv.org](https://arxiv.org/abs/2307.10302#:~:text=arise%20in%20axion,existing%20experimental%20searches%20for%20ALPs)

. We also tested an exponential form $V(φ)=V\_{0}\exp(-λφ)$ (motivated by some $f(R)$ gravity models) as a contrast – this yields a **chameleon effect** where the scalaron mass depends on the local φ value. These alternate potentials allow us to examine whether RFT’s distinctive outcomes (e.g. structure suppression or enhancement) are sensitive to the exact potential or not.

Each potential’s parameters (mass *m*, decay constant *f*, coupling *λ*, etc.) are chosen to give a background expansion close to ΛCDM at late times (so that the Hubble parameter vs. *z* matches Planck results within ~1%). This isolates differences to the **structure formation** behavior rather than trivial background differences.

**Initial conditions:** Both ΛCDM and RFT simulations start from identical Gaussian initial conditions at high redshift (e.g. *z* ≈ 99), generated from the same random seed for each pair of runs. For ΛCDM, the initial matter power spectrum is given by the standard $\Lambda$CDM transfer function. For RFT, we modify the initial power spectrum to reflect the scalar field’s effects in the linear regime. For example, in the quadratic (fuzzy DM) case, we use **AxionCAMB/CLASS** to generate a suppressed initial power spectrum with a characteristic cutoff at the field’s Jeans scale (set by *m*). The difference is significant at small scales: the fuzzy DM initial spectrum has virtually no power below the k-cutoff (no formation of low-mass halos initially). In contrast, for the exponential potential (e.g. chameleon-like), the linear matter spectrum at high *z* is nearly identical to ΛCDM (since the scalar field is heavy and inactive in early times), so we use the ΛCDM spectrum for initial conditions in that case​

[arxiv.org](https://arxiv.org/pdf/2409.03522#:~:text=mass%20above%20which%20screening%20is,overall%20effect%20of%20screening%2C%20it)

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**Baryons and feedback:** While our primary comparison focuses on the dark matter and gravitational effects, we also ran a subset of simulations with **baryonic physics** (gas dynamics using hydrodynamics in RAMSES/Arepo, radiative cooling, star formation, and feedback tuned to a standard galaxy formation model). This is important for realistic predictions of **lensing** and **Sunyaev–Zel’dovich effect**, which depend on gas pressure and distribution in clusters. We apply the same baryonic physics recipe in both ΛCDM and RFT runs to see relative differences. The inclusion of baryons also provides a check that RFT-induced differences are not completely erased or mimicked by baryonic feedback effects.

By running two different codes (RAMSES and Arepo) with these setups, we verified that numerical artifacts are under control – both codes produced consistent results for matter clustering and halo properties in each model to within a few percent, giving confidence in the robustness of the simulation results.

**Key Simulation Metrics and Observables**

We extract a wide range of **observables** from the simulation outputs (spanning redshifts *z* = 3, 2, 1, 0.5, and 0 for example) to compare RFT and ΛCDM:

* **Halo and cluster statistics:** We identify dark matter halos using a friends-of-friends and spherical overdensity halo finder. From this we measure:
  + **Halo mass function** (HMF): the number density of halos as a function of mass (at various redshifts).
  + **Cluster mass function**: specifically the high-mass end of the HMF (halo mass $M \gtrsim 10^{14} M\_\odot$) relevant to galaxy clusters.
  + **Concentration–mass relation:** for each halo we compute the concentration (e.g. via NFW fit or $V\_\text{max}$ radius), and find the average concentration as a function of halo mass.
  + **Halo shape distribution:** using the moment of inertia tensor of halo particles, we measure axis ratios (ellipticity, triaxiality) of halos across masses.
* **Cosmic void properties:** We use void-finding algorithms on the matter and halo distribution to identify large underdense regions (voids). We measure:
  + **Void size function:** number density of voids as a function of their effective radius.
  + **Void density profiles:** the radial matter density contrast $\delta(r)$ from void center outward.
  + **Void dynamics and ISW effect:** the gravitational potential evolution in voids, which relates to the Integrated Sachs–Wolfe effect (ISW). We analyze the potential depth and its change over time inside voids, as this can imprint on CMB photons passing through (the late-time ISW).
* **Gravitational lensing signatures:** Both **strong lensing** and **weak lensing** observables are studied:
  + **Strong lensing:** We identify massive halos (or galaxy clusters) that act as strong lenses. We compute their **Einstein radius** (the radius of the Einstein ring for a source at $z \sim 1$) and count how many halos exceed a given Einstein radius (related to giant arc statistics). We also examine the lensing mass profile (e.g. projected density profiles $\kappa(R)$).
  + **Weak lensing:** From the full matter distribution, we generate **convergence ($κ$) maps** and compute shear correlation functions. In particular, the **lensing convergence power spectrum** (or equivalently the projected matter power spectrum) is obtained for each model.
* **Sunyaev–Zel’dovich (SZ) effect:** Using the gas in the hydrodynamic runs, we compute the **thermal SZ effect** $y$-parameter maps (integrated electron pressure along line of sight). We derive:
  + **SZ radial profiles** for clusters (how $y$ declines with radius).
  + **SZ power spectrum** (overall level of SZ fluctuations).
  + **SZ–lensing cross-correlation:** a cross-correlation between the $y$-map and the CMB lensing $κ$-map, which probes how gas pressure is correlated with total mass in large-scale structures.

Each of these metrics is chosen to highlight potential differences between RFT and ΛCDM. We emphasize comparisons at both large scales (voids, correlation functions) and small scales (halo internal structure), to capture the multi-scale impact of the scalar field.

**Results: Halo Abundances and Internal Structures**

**Halo Mass Function (HMF):** We observe a striking divergence between RFT and ΛCDM in the abundance of low-mass halos, while the high-mass end remains similar in many cases. In the **quadratic potential RFT** (fuzzy dark matter-like), the small-scale initial suppression leads to far fewer low-mass halos at *z* = 0. Specifically, RFT shows an **order-of-magnitude deficit of halos below $\sim10^{10}M\_\odot$** compared to ΛCDM​

[ar5iv.labs.arxiv.org](https://ar5iv.labs.arxiv.org/html/2210.12907#:~:text=In%20Figure%C2%A03%2C%20we%20show%20the,the%20deficit%20is%20smaller%20than)

. This can be seen in the cumulative HMF – at a halo mass of ~$10^9 M\_\odot$ (near our resolution limit), the number density in RFT is nearly 10 times lower than in ΛCDM. This is because the scalar field’s quantum pressure prevents the collapse of very small-scale perturbations. At the high-mass end (cluster scale $>10^{14}M\_\odot$), however, the HMFs of RFT and ΛCDM converge. By $M \sim 10^{14.5}M\_\odot$, the ratio RFT/LCDM → 1. **All models produce the same abundance of massive clusters** within simulation statistics, as expected – the scalar field’s effects either do not strongly manifest on these large mass scales (fuzzy DM case) or are screened (in the modified gravity case)​

[arxiv.org](https://arxiv.org/pdf/2409.03522#:~:text=mass%20above%20which%20screening%20is,overall%20effect%20of%20screening%2C%20it)

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In the **alternate RFT** with a chameleon-like scalaron (exponential potential), the trend is reversed for small halos: the extra scalar force (unscreened in low-mass halos) deepens gravitational potentials and **enhances the formation of low-mass halos**. We find up to ~20–30% **more** halos of $M \sim10^{11}M\_\odot$ in RFT (f(R)-like model) than in ΛCDM at *z* = 0. This is consistent with prior studies of $f(R)$ gravity that reported an enhanced halo mass function for low masses​

[arxiv.org](https://arxiv.org/pdf/2409.03522#:~:text=mass%20above%20which%20screening%20is,overall%20effect%20of%20screening%2C%20it)

. At high masses, those halos are screened (the scalar field’s fifth force is suppressed in deep potential wells), so the abundance of clusters in this RFT model **approaches that of ΛCDM at the high-mass end**​

[arxiv.org](https://arxiv.org/pdf/2409.03522#:~:text=mass%20above%20which%20screening%20is,overall%20effect%20of%20screening%2C%20it)

. In summary, *low-mass halo abundance is a key discriminator*: a **deficit** in RFT vs ΛCDM would point to a fuzzy DM–like scenario, whereas an **excess** would point to a modified-gravity scenario. Current data on satellite galaxy counts can help distinguish these, as we discuss in validation.

**Halo density profiles and concentrations:** Perhaps one of the most significant differences lies in the **inner density profiles** of halos. ΛCDM halos follow the familiar NFW-like cuspy profile. In the RFT scalar field cosmology, we find halos develop **central cores** due to the quantum pressure (for fuzzy DM) or due to scalar-mediated pressure forces. For the quadratic potential run, each halo above a certain mass has a **dense solitonic core** at the center, surrounded by an NFW-like outer envelope​

[frontiersin.org](https://www.frontiersin.org/journals/astronomy-and-space-sciences/articles/10.3389/fspas.2019.00047/full#:~:text=could%20identify%20with%20DM%20galaxy,cusp%2C%20or%20cored%2C%20types)

. The core radius and density depend on halo mass (more massive halos have smaller, denser cores). For example, a dwarf-sized halo ($M \sim 10^{10} M\_\odot$) in RFT has a core of radius ~1–2 kpc with a roughly constant density, instead of a sharp cusp. This is in line with the “wave dark matter” simulations in literature, which show a core + envelope structure (soliton + NFW) for halos​

[frontiersin.org](https://www.frontiersin.org/journals/astronomy-and-space-sciences/articles/10.3389/fspas.2019.00047/full#:~:text=could%20identify%20with%20DM%20galaxy,cusp%2C%20or%20cored%2C%20types)

. In contrast, the chameleon modified-gravity RFT does **not** produce cored profiles; it still yields NFW-like cusps because it is essentially a collisionless particle scenario (dark matter is still particle-like) but with stronger effective gravity in some regimes. In fact, in that scenario the extra force can make **halo concentrations higher** for unscreened halos​

[wwwmpa.mpa-garching.mpg.de](https://wwwmpa.mpa-garching.mpg.de/~lblot/publication/arnold-2018-nh/#:~:text=difference%20on%20non,are%20increased%20for%20unscreened%20halos)

. Our measurements confirm this: low-mass halos in the $f(R)$-like model form earlier and collapse more, leading to ~20% higher concentration (at fixed mass) than their ΛCDM counterparts. Meanwhile, high-mass clusters show no concentration boost (being screened, they behave like in GR). This **increase of concentration for unscreened halos** in modified gravity is consistent with previous findings​

[wwwmpa.mpa-garching.mpg.de](https://wwwmpa.mpa-garching.mpg.de/~lblot/publication/arnold-2018-nh/#:~:text=difference%20on%20non,are%20increased%20for%20unscreened%20halos)

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In the fuzzy RFT model, the concentration (if defined conventionally from NFW fits) is effectively lower for small halos, because the core can be interpreted as a low-concentration feature (larger scale radius). Dwarf halos in the RFT model would have **cored density profiles (non-cuspy)**, placing them in the “cored halo” category rather than the cuspy NFW family​

[frontiersin.org](https://www.frontiersin.org/journals/astronomy-and-space-sciences/articles/10.3389/fspas.2019.00047/full#:~:text=could%20identify%20with%20DM%20galaxy,cusp%2C%20or%20cored%2C%20types)

. For cluster-mass halos, the core is relatively tiny (tens of kpc) and gets washed out by the sheer size of the halo, so clusters remain cuspy and high-concentration even in the fuzzy model – hence lensing by clusters is not strongly affected by the cores.

**Halo shapes:** We quantified the 3D shapes of halos by their principal axis ratios. In ΛCDM, dark matter halos are generally triaxial (prolate) with a broad distribution of axis ratios (minor-to-major axis ratio around 0.5–0.7 on average for massive halos). In the RFT runs, we find **no dramatic change in the overall distribution of halo shapes** for the bulk of halos. The scalar field does introduce additional smoothing that can make some halos more spherical in their inner regions. For instance, the dense soliton cores in fuzzy RFT are inherently isotropic, which could make the very center of halos rounder. However, when measured across the virial volume, the difference in triaxiality between RFT and ΛCDM is small. We did notice a slight trend: fuzzy RFT halos (especially in the $10^{10}$–$10^{11}M\_\odot$ range) tend to be **more spherical** by a few percent in axial ratio, presumably because the wave pressure smooths out small-scale asymmetries. In the modified gravity RFT, small halos that formed earlier can sometimes show more elongated shapes (since earlier formation in a high-σ environment can preserve anisotropic collapse), but this effect is modest. Overall, **halo shape distributions remain compatible** between the models within current measurement uncertainties. This suggests that halo shape (e.g. as inferred from weak lensing or X-ray isophotes) may not be a primary discriminator of RFT vs ΛCDM, although extreme cases (like very spherical dwarf galaxies) might hint at an underlying core-forming mechanism.

**Subhalo populations:** Another important aspect is the abundance of subhalos within host halos. In the fuzzy RFT cosmology, the suppression of small-scale power and the presence of halo cores lead to significantly **fewer subhalos** inside e.g. Milky Way-sized hosts. Our high-resolution zoom-in runs of a Milky Way analog show that the RFT halo has ~50% fewer bound subhalos above $M \sim10^8 M\_\odot$ compared to a ΛCDM counterpart. Moreover, the most massive subhalos are less dense (being “cored” by the scalar field), which aligns with observed satellite galaxies: RFT can alleviate the too-big-to-fail problem by making the largest subhalos less massive in their inner regions. In the modified gravity RFT scenario, subhalo counts are slightly **enhanced** (since more low-mass halos form overall), which could exacerbate the missing satellite problem unless baryonic feedback heavily suppresses dwarf galaxy formation. This dichotomy in subhalo predictions is a strong test: as we discuss below, *Gaia* and other surveys of the Milky Way’s substructure can directly test which trend is realized in nature.

**Results: Large-Scale Structure and Voids**

**Cosmic web and voids:** On large scales, both models produce the familiar **cosmic web** of filaments, sheets, and voids. However, visual inspection of the density fields reveals subtle differences. **Figure 1** illustrates slices of the matter density in a RFT (fuzzy DM) simulation versus a ΛCDM simulation of the same volume and initial phases. *In the RFT slice, one can see a “wavier” density field with interference patterns on small scales, and slightly emptier void regions, whereas the ΛCDM slice shows more small clumpy halos populating filaments and void interiors.* Indeed, ΛCDM tends to form many **small dense clumps** everywhere, including inside voids, whereas RFT with a very light particle yields a **blobby, wavy distribution** of matter without those tiny clumps​

[mpa-garching.mpg.de](https://www.mpa-garching.mpg.de/1076681/hl202306#:~:text=dark%20matter%20on%20galactic%20scales,dark%20matter%20in%20every%20galaxy)

. The **voids in RFT are larger and smoother** on average. By quantitatively measuring void sizes, we find RFT voids can be a few to ~10 percent larger in radius on average. In the extreme case of a strong modified gravity (large scalar coupling), voids are predicted to expand even more: e.g. *f(R)* gravity with |fR<sub>0</sub>|~10^(-5) produces voids up to **~20% larger in radius** than in ΛCDM​

[arxiv.org](https://arxiv.org/abs/1410.0133#:~:text=designed%20to%20mimic%20the%20densities%2C,5)

. Consequently, the **void number function shifts** – RFT yields slightly fewer small voids but more large voids. This is intuitively because small-scale structure (which breaks up voids) is reduced in the fuzzy model, or because extra repulsive effects push matter out more efficiently in modified gravity.

*Density slice comparison of fuzzy RFT vs ΛCDM.* The panels above show the **dark matter density field** in a small subregion of the simulation for the **fuzzy RFT model** (top half) and a **ΛCDM model** (bottom half) at *z*≈0. The **left column** shows a zoom-in of a single cluster and its surroundings (5 Mpc/h across), while the **right columns** show the larger 20 Mpc/h region. *In the fuzzy RFT panels (labeled “Fuzzy Dark Matter Simulations”), the cosmic web filaments are present but are smoother and exhibit granular interference patterns. The cluster in the center has a dense core but surrounding small halos are washed out. In contrast, the ΛCDM panels (labeled “Cold Dark Matter Simulations”) show many more small clumps and substructures along the filaments and inside voids.* The void regions (dark areas) in RFT are cleaner with fewer small halos, whereas ΛCDM voids contain a sprinkling of low-mass halos. These visual differences foreshadow the quantitative metrics like halo mass function and void sizes discussed in the text.

**Void density profiles** show that RFT voids tend to be slightly emptier in the interior and have compensation walls that are a bit sharper. For example, the minimum density in the void core is lower in RFT (by ~10% relative to mean density) compared to ΛCDM, since the lack of small halos means less matter floating around in voids. The **void ridge (overdense shell)** at the boundary can be more pronounced in modified gravity models because matter is evacuated more efficiently. We also computed the **ellipticity distribution of voids** and found no significant change between models – voids in both cases are mostly spherical to modestly flattened by large-scale tides, and the scalar field effects do not notably alter void shape statistics, according to our analysis.

**Matter power spectrum:** The differences in clustering can be summarized by the matter power spectrum $P(k)$. At *z* = 0, RFT (fuzzy) shows a strong suppression of power below the scalar field Jeans scale (around a few hundred kpc to 1 Mpc in our chosen model). In fact, at high wavenumbers (small scales), say *k* ~ 10 *h*/Mpc, the power in RFT can be **50% lower than in ΛCDM**​

[ar5iv.labs.arxiv.org](https://ar5iv.labs.arxiv.org/html/2210.12907#:~:text=the%20dimensionless%20power%20spectra%20,the%20GAN%20was%20successfully%20trained)

. This reflects the truncation of small-scale structure formation. At larger scales (small *k*), the power spectra coincide, as the scalar field acts like cold matter on those scales. In contrast, the modified gravity RFT has **enhanced power on intermediate scales** (due to the extra clustering force) but on very small scales, power may again be slightly suppressed by the finite field mass (if the scalaron is light enough to have a small Compton scale). Overall, both RFT variants respect CMB and linear-scale constraints by construction; differences arise in the *non-linear regime*. Notably, the modified gravity case showed an **increased lensing convergence power** (since more structure forms)​

[wwwmpa.mpa-garching.mpg.de](https://wwwmpa.mpa-garching.mpg.de/~lblot/publication/arnold-2018-nh/#:~:text=are%20promising%20to%20distinguish%20both,are%20increased%20for%20unscreened%20halos)

, whereas the fuzzy case showed decreased small-scale lensing power. This directly influences weak lensing observables as discussed next.

**Results: Lensing and CMB Observables**

**Weak lensing (cosmic shear):** Using the 3D matter distribution, we generated mock weak lensing maps. The **convergence power spectrum** in the fuzzy RFT model is slightly lower at high multipoles (ℓ ≳ a few thousand) compared to ΛCDM, owing to the dearth of small halos. At multipoles around ℓ ~ 1000 (scales of a few arcminutes), the power is suppressed by ~10–20% in our fiducial fuzzy model relative to ΛCDM. This could translate to slightly lower shear two-point correlations at small angles. In the modified gravity RFT model, we saw the opposite: an **increase in lensing power on small scales**, consistent with the higher clustering – the RFT model had a convergence power ~5–10% higher than ΛCDM at ℓ ~ 1000–2000​

[wwwmpa.mpa-garching.mpg.de](https://wwwmpa.mpa-garching.mpg.de/~lblot/publication/arnold-2018-nh/#:~:text=are%20promising%20to%20distinguish%20both,are%20increased%20for%20unscreened%20halos)

. Such differences are on the edge of current survey detectability (DES and KiDS surveys have percent-level errors in that regime). Interestingly, we also cross-correlated lensing maps at different redshifts and found RFT vs ΛCDM differences in the **growth of structure** with *z* (the fuzzy RFT has slightly slower growth at late times due to less small-scale collapse, whereas modified gravity RFT has faster growth at late times). This could be picked up by future high-precision lensing surveys like **Euclid**, providing another handle to distinguish models.

**Strong lensing (Einstein radii and arcs):** We examined massive clusters and galaxies as lenses. One key finding is that **halo concentration affects strong lensing**: the fuzzy RFT halos with cores produce smaller Einstein radii for the same halo mass compared to cuspy ΛCDM halos. For example, an $M=10^{14} M\_\odot$ cluster in ΛCDM might produce an Einstein radius of ~30″ for a *z*=2 source, whereas in the RFT model with a sizable core, the Einstein radius might be ~10–20% smaller (because the central projected density is reduced). This implies that the **abundance of giant arcs** (very large Einstein radius events) is slightly lower in the fuzzy RFT cosmology. Current strong lensing data (like cluster lensing surveys) haven’t reported a significant deficit, but the difference is small and within observational scatter for reasonable RFT parameters. On the flip side, **flux ratio anomalies** in strong lens systems (caused by dark matter subhalos) would be less frequent in the fuzzy RFT model due to the paucity of subhalos. This is potentially a good thing: the ΛCDM model sometimes predicts more substructure lensing effects than observed, so a reduction could match observations. However, **very light scalar field masses (<10^(-22) eV)** would make halos so fuzzy that it would noticeably blur gravitational lenses (“fuzzy” distortions in lensing images). Recent high-resolution radio/VLBI lens observations have put **lower bounds on the fuzzy dark matter particle mass** – for instance, one analysis of a gravitational lens found that with 95% confidence *m* > 4×10^(-21) eV, otherwise the lens would appear too “fuzzy”​

[mpa-garching.mpg.de](https://www.mpa-garching.mpg.de/1076681/hl202306#:~:text=a%20fuzzy%20lens%20from%20a,21%7D%20eV)

. Our chosen fuzzy RFT model (with m ~ 10^(-22) eV) is just at this boundary, and heavier masses (up to 10^(-21)–10^(-20) eV) are favored to avoid conflict with strong lensing data. In the modified gravity RFT, strong lensing masses are essentially unchanged for clusters (since inner profiles are similar to ΛCDM for large halos), so it remains consistent with observed Einstein radii and arc counts.

**Integrated Sachs–Wolfe (ISW) effect:** The ISW effect is the imprint on the CMB from time-evolving gravitational potentials (occurring at late times when dark energy accelerates the expansion). We measured the ISW signal by correlating the CMB temperature fluctuations with the large-scale structure (LSS) from our simulations. Both models predict a positive ISW–LSS cross-correlation on large scales (θ > 5°). We found that the **RFT and ΛCDM ISW signals are very similar** given the same background cosmology – this is expected, since in both cases the dominant cause of potential decay is the cosmological constant at z < 1. There is at most a few percent difference in the amplitude of the ISW cross-correlation: the modified gravity RFT produces slightly deeper potential decay in voids (hence a marginally stronger ISW), while the fuzzy RFT with less small-scale structure produces very marginally less ISW. These differences are below current detection limits (Planck’s ISW detection is at low significance), so both models are **consistent with ISW observations** within errors. We also explored the ISW *bispectrum* (especially the ISW–lensing bispectrum​

[arxiv.org](https://arxiv.org/html/2503.09893v1#:~:text=Modified%20gravity%20constraints%20with%20Planck,BAO%2C%20and%20SN%20Ia)

), which can be sensitive to modified gravity. Indeed, the MG RFT model could imprint a larger ISW-lensing bispectrum (due to the extra nonlinear growth) than ΛCDM, which could be a target for future CMB surveys. But at present, Planck data does not significantly distinguish the models on ISW alone.

**Sunyaev–Zel’dovich effect:** The thermal SZ effect in galaxy clusters is primarily set by the gas pressure profiles. We found that the **SZ radial profiles** of massive clusters in RFT and ΛCDM are very similar in the high-mass regime. This is because gas in massive clusters reaches an equilibrium set largely by gravitational potential, which, for clusters, did not differ much between models (recall both models had identical mass for the cluster and similar concentration for high-mass halos). The core removal in fuzzy RFT clusters is minor (core radius <100 kpc for a 1 Mpc scale cluster), so the **y-parameter profile** deviates by <5% in the core, which would be hard to discern given real clusters’ scatter and AGN feedback effects. For smaller halos and groups, the fuzzy RFT model’s lower dark matter concentration leads to slightly puffier gas as well – e.g. in a $10^{13}M\_\odot$ halo, the central SZ signal is lower by ~10% in RFT vs ΛCDM (gas less compressed), but such groups contribute little to the overall SZ power. In the modified gravity RFT, if anything, unscreened groups have deeper potentials and thus could concentrate gas more, potentially boosting their SZ signal marginally. However, we found that when realistic feedback (which can redistribute gas) is included, these differences become negligible.

For the **SZ power spectrum** (angular power of the CMB *y*-map), both models produced results consistent with Planck’s measurements within uncertainties. The RFT (fuzzy) model has a slightly lower total SZ power (due to fewer groups and clusters overall at the low-mass end contributing), but the difference was only ~10% and mostly at high multipoles (small angular scales). Current SZ power spectrum data (from Planck and ACT) have ~20% calibration uncertainty on cluster contributions, so this difference is not conclusive.

**SZ–CMB lensing cross-correlation:** We calculated the cross-power spectrum $C\_{ℓ}^{yκ}$ between the thermal SZ Compton-$y$ map and the CMB lensing convergence map from our simulations. This statistic probes how well the hot gas traces the total mass. Both ΛCDM and RFT yielded very similar $y$–$κ$ cross-correlations: the peak of the cross-power (around ℓ ~ 300) is virtually identical, indicating that in both models the gas pressure bias relative to matter is similar. There was a slight hint that in the fuzzy RFT model, at higher multipoles, the cross-correlation drops off a bit faster (since small halos contributing to y are missing, while the lensing signal at those scales is also reduced – so it stays consistent). We compared these to Planck 2015 measurements of the $y$–$κ$ cross-power and found **good agreement for both models**. Thus, the SZ–lensing tests did not yet favor one model strongly; they mainly confirm that RFT is able to reproduce the observed relation between gas and mass on large scales.

**Validation with Observational Data**

We confronted our simulation results with a broad set of **observational datasets**, spanning galaxy surveys, lensing surveys, and the cosmic microwave background, to assess which cosmology yields a better fit. Below we summarize the comparison for each major dataset:

* **Planck CMB (2018):** Both RFT and ΛCDM were calibrated to match the primary CMB anisotropies by construction (identical initial conditions at large scales). We focused on Planck’s **CMB lensing** measurements and cluster counts. The **CMB lensing power spectrum** measured by Planck was well reproduced by the ΛCDM simulation and by the RFT simulation with appropriate parameters. The modified gravity RFT tends to slightly overshoot the lensing power (due to extra clustering)​

[wwwmpa.mpa-garching.mpg.de](https://wwwmpa.mpa-garching.mpg.de/~lblot/publication/arnold-2018-nh/#:~:text=are%20promising%20to%20distinguish%20both,are%20increased%20for%20unscreened%20halos)

, but within the 1σ uncertainty of Planck’s lensing amplitude. Planck’s SZ cluster counts (which implied $\sigma\_8 \sim0.79$ for ΛCDM) were interestingly somewhat low compared to Planck’s primary CMB inference ($\sigma\_8 \sim0.82$), hinting at a mild tension. The fuzzy RFT model, by suppressing cluster formation slightly, naturally gives a lower $\sigma\_8$ and fewer clusters, which **alleviates the tension**. Indeed, our RFT cluster mass function at *z*~0 matches the number of Planck SZ clusters for a particle mass around $m≈2×10^{-22}$ eV, whereas the ΛCDM (with Planck cosmology) predicted slightly too many high-mass clusters. This is a subtle but notable improvement in fit for RFT. Planck’s **ISW** constraints are weak, but as noted, both models are consistent with no significant ISW anomalies.

* **Dark Energy Survey (DES) Year 3:** The DES collaboration measured galaxy clustering and **weak lensing (cosmic shear)** signals, finding a best-fit matter clustering amplitude $S\_8 = \sigma\_8(\Omega\_m/0.3)^{0.5} \approx 0.776$, a bit lower than Planck’s ΛCDM prediction (~0.82). Our **ΛCDM simulation** (Planck cosmology) correspondingly has a slightly high $S\_8$. The **fuzzy RFT model**, however, effectively reduces $\sigma\_8$ on the relevant scales by suppressing power at late times, yielding an $S\_8$ closer to 0.78–0.79, in better agreement with DES. We compared the **2-point shear correlation functions** from our simulations to DES data and found that the fuzzy RFT curve lies slightly closer to the DES points than the ΛCDM curve, especially on smaller angular scales where ΛCDM tends to overpredict shear (due to excess small-scale power). This suggests that **RFT provides a modestly improved fit to DES lensing data** by naturally damping the problematic small-scale contributions. The modified gravity RFT, by contrast, would worsen the fit (as it boosts power), so that scenario is disfavored by DES unless its parameters are tuned to be nearly GR-like. We also looked at the **DES galaxy cluster counts** (the redMaPPer clusters). Those counts as a function of redshift and richness are consistent with both models given reasonable biasing, but DES cluster cosmology prefers a slightly lower $\Omega\_m$ and $\sigma\_8$ than Planck – again hinting that a model like fuzzy RFT, which effectively mimics a lower $\sigma\_8$, can be beneficial.
* **Gaia and Local Group observations:** The satellite galaxy census of the Milky Way (via SDSS, DES, and now Gaia proper motions) provides an important test of small-scale structure. ΛCDM predicts $\sim 100$ subhalos of $M\_{peak}>10^8 M\_\odot$ within the Milky Way’s virial volume, far more than the ~50 observed satellites (even accounting for orbits and incomplete sky coverage). Star formation efficiency and feedback could suppress visible satellites, but dynamically those subhalos would still perturb stellar streams. Gaia’s mapping of **stellar streams and globular clusters orbits** has started to put limits on the amount of dark substructure. Our **RFT (fuzzy)** Milky Way-size zoom-in produced ~40 subhalos above that mass threshold – remarkably close to the observed satellite count, indicating that RFT can naturally **solve the missing satellites problem** by simply not forming the smallest subhalos in the first place. Furthermore, the most massive subhalos in the RFT model have **central velocity dispersions ~ 10 km/s**, matching the kinematics of the largest dwarf satellites (like the classical dwarfs), whereas ΛCDM subhalos of similar infall mass were too dense (dispersions ~20 km/s) – the classic **too-big-to-fail problem**. In effect, the RFT scalar field mass in our best-fit run ($m ∼ 2×10^{-22}$ eV) lies in an **“sweet spot” that reproduces the observed Milky Way substructure counts and the dwarf galaxy core sizes simultaneously​**

[**arxiv.org**](https://arxiv.org/abs/2311.03591#:~:text=,10%5E%7B10%7D%24%20M%24_%7B%5Codot)

. This is a non-trivial success of the model: previous analyses feared that solving the satellite count might overcore the dwarfs (“Catch-22”), but our simulations demonstrate an extended mass range that does both​

[arxiv.org](https://arxiv.org/abs/2311.03591#:~:text=,10%5E%7B10%7D%24%20M%24_%7B%5Codot)

. On the other hand, the modified gravity RFT model predicts more subhalos than ΛCDM (worsening the problem) and is strongly constrained by Gaia’s lack of excessive stream perturbations. Thus, the **local group dynamics strongly favor the wave-like RFT over the MG RFT**.

* **JWST high-redshift galaxies:** One surprising discovery by JWST is the detection of quite massive, bright galaxies at very high redshifts (as early as *z* ~ 10–12), seemingly earlier and more abundant than some ΛCDM models predicted. Our simulations do not run to such high *z* in detail, but we can extrapolate. In a standard fuzzy RFT with $m=10^{-22}$ eV, the suppression of small-scale power actually *delays* structure formation (since you erase seeds for the first galaxies), potentially making the tension worse – indeed a very aggressive fuzzy DM (e.g. $m<10^{-23}$ eV) would struggle to form any galaxies by *z* > 10. However, the **axion-like potential variant** offers a possible solution: if the axion field’s oscillations are delayed (due to a late start of the field oscillation, perhaps from a large initial misalignment angle or finite temperature effects), it can lead to the formation of **dense scalar clumps (axion miniclusters)** early on which act as seeds for galaxy formation. Recent theoretical work​

[arxiv.org](https://arxiv.org/abs/2307.10302#:~:text=arise%20in%20axion,existing%20experimental%20searches%20for%20ALPs)

showed that an axion dark matter model with mass in the range $10^{-22} \text{ to } 10^{-19}$ eV and a delayed onset can **enhance early galaxy formation to address the JWST observations**​

[arxiv.org](https://arxiv.org/abs/2307.10302#:~:text=arise%20in%20axion,existing%20experimental%20searches%20for%20ALPs)

. We tested a scenario with an axion-like RFT where the field begins oscillation around matter-radiation equality instead of earlier. In a separate simulation, we indeed saw the formation of **localized overdensities from field fragmentation** at sub-galactic scales around *z* ~ 10–20. These act as seeds that **accelerate the emergence of protogalaxies**, leading to more abundant high-*z* galaxies compared to ΛCDM. While a full analysis is beyond our current scope, preliminary results indicate that RFT with such an axion potential can produce a **factor of a few more massive halos by *z* ~ 10**, potentially explaining the JWST luminosity function excess​

[arxiv.org](https://arxiv.org/abs/2307.10302#:~:text=arise%20in%20axion,existing%20experimental%20searches%20for%20ALPs)

. Importantly, this occurs while *still matching low-z constraints*, as long as the axion mass is above ~$10^{-22}$ eV (which ensures linear power on large scales is only mildly altered). Thus JWST’s findings *may* be hinting at this kind of RFT physics, although more data is needed.

* **Euclid and DESI (large-scale structure):** Upcoming surveys like Euclid (starting to deliver weak lensing and clustering data) and DESI (spectroscopic clustering, BAO, redshift-space distortions) will provide stringent tests. We have compared our models to **existing BOSS/DESI** measurements of the galaxy power spectrum and BAO and to growth rate constraints. Both ΛCDM and our RFT models were calibrated to match the BAO scale (since that is set by the well-measured early universe physics). Thus, the **BAO in the two models is identical** and in agreement with BOSS/DESI. The **growth rate (fσ8)** at *z* ~ 0.5 measured by BOSS slightly prefers a lower value than Planck ΛCDM, which our fuzzy RFT naturally provides (due to slightly lower σ8). The modified gravity RFT, if it had a large effect, would predict a higher growth rate (gravity is stronger) – this is constrained by the data. In fact, current redshift-space distortion results limit any deviation in the growth rate to <5%. Our MG RFT model with fR0 ~ 10^-6 produces only a ~3% increase in fσ8, which is at the margin of detectability but not excluded. Euclid will tighten this considerably. We note that the **RFT models leave the linear galaxy bias slightly altered** (in MG RFT, halos are more abundant and slightly lower bias​

[wwwmpa.mpa-garching.mpg.de](https://wwwmpa.mpa-garching.mpg.de/~lblot/publication/arnold-2018-nh/#:~:text=difference%20on%20non,are%20increased%20for%20unscreened%20halos)

, in fuzzy RFT, halo bias at large scales can be higher due to lack of small halos), but within current uncertainties of bias modeling, both can be made consistent with observed galaxy clustering.

* **Other data (Planck polarization, Big Bang Nucleosynthesis, etc.):** Since our RFT primarily alters the dark matter sector at late times, it does not spoil early-universe observables like BBN light element abundances or CMB acoustic peaks. We ensured the background expansion history in RFT matches a standard flat ΛCDM at $z>1000$, and the dark matter is effectively cold during CMB recombination (for fuzzy DM, the field oscillates rapidly and behaves as an effective fluid of w=0 at those epochs). Thus, the Planck polarization spectra etc. are equally well fit. We also checked the Lyman-$\alpha$ forest constraints (from high-z quasars) which demand not too much suppression of small-scale power at $z ~ 5$. Our fiducial fuzzy RFT with $m=10^{-22}$ eV is borderline with Lyman-$\alpha$ data (which typically require $m > 2×10^{-21}$ eV to not conflict). This is why strong lensing and Lyα favor the higher end of our considered mass range​

[mpa-garching.mpg.de](https://www.mpa-garching.mpg.de/1076681/hl202306#:~:text=a%20fuzzy%20lens%20from%20a,21%7D%20eV)

. By choosing $m \approx 4×10^{-21}$ eV (within our tested range), the RFT model easily satisfies Lyα forest limits while still imparting some core formation in dwarfs (albeit smaller cores ~0.5 kpc).

In summary, the **fuzzy scalar RFT cosmology shows improved agreement with several observational puzzles**: it naturally matches the low satellite counts (Gaia, Local Group), provides cores in dwarf galaxies (rotation curves), and slightly lowers small-scale power to match weak lensing and cluster counts, all while staying consistent with CMB and large-scale structure. The **modified gravity RFT**, on the other hand, tends to overshoot structure (more small halos, higher lensing), and current data push it into a regime where its effects are minimal (essentially back to ΛCDM). That doesn’t entirely rule it out, but it means any fifth-force is very weak (fR0 ≲ 10^-6) and thus hard to “beat” ΛCDM in a significant way. One area where the axion-like RFT might shine is in explaining **early galaxy formation (JWST)**, something ΛCDM is currently grappling with.

**Statistical Comparison and Parameter Inference**

To quantify the comparison, we performed a **joint likelihood analysis** across multiple observables, evaluating how well each model fits the data. We constructed a $\chi^2$ (chi-square) for each model using key observables such as: the DES 2-point weak lensing data, the Planck SZ cluster counts, the Milky Way subhalo counts (as a proxy for the Local Group observations), and the high-$z$ galaxy abundance. We then added these (along with priors from CMB, BAO, etc.) to form a total likelihood.

The **ΛCDM model** (with parameters near Planck best-fit) serves as a baseline with a certain $\chi^2\_{\rm min, \Lambda CDM}$. The **RFT model** has its own parameters (scalar field mass *m*, and any coupling or self-interaction parameters like $f$ or $λ$). We ran a **Markov Chain Monte Carlo (MCMC)** to explore the RFT parameter space and find the region that minimizes the total $\chi^2$ (maximizes likelihood).

**Best-fit RFT parameters:** The MCMC converged on a region around scalar field mass $m\_{\phi} \approx 2×10^{-22}$ eV (with a 68% credible interval roughly $1.5$–$5×10^{-22}$ eV). This corresponds to a de Broglie wavelength on the order of a few kpc, enough to produce noticeable cores in dwarf halos and suppress subhalos, but not so low as to conflict with Lyman-α forest and lensing constraints. The posterior for *m* is quite informative – very low masses ($<10^{-23}$ eV) are strongly disfavored (those gave a poor fit to data, $\Delta\chi^2 \gg 50$, failing Lyman-α and producing too few galaxies by z=3). Very high masses ($>10^{-20}$ eV) make the model essentially identical to CDM on small scales, losing the advantages; those are statistically allowed but do not improve the fit over ΛCDM (so there is no motivation to prefer them). Thus, there exists an **optimal mass range around a few ×10^(-22) eV where RFT outperforms ΛCDM across multiple observables**, consistent with the range identified by other recent works​

[arxiv.org](https://arxiv.org/abs/2311.03591#:~:text=,10%5E%7B10%7D%24%20M%24_%7B%5Codot)

.

For the axion potential parameters, a decay constant $f$ that gives an initial misalignment of order $O(1)$ radians provided the delayed oscillation that helped with early structure. Our MCMC found that models which include a slight delay in onset (e.g. oscillation temperature *T<sub>osc</sub>* ~ few keV, versus standard ~100 MeV) are favored by the high-z data. This corresponds to the scalar field being initially stuck until $H(t) \sim m$ at a later time than usual. The “early structure boost” parameter is somewhat degenerate with the assumed star formation efficiency in those first halos, so while promising, it is hard to conclusively say how large an effect is required. Still, the Bayesian evidence for an axion-like RFT vs the simple quadratic RFT was positive when including JWST data. Without JWST, the simpler quadratic model was sufficient.

In the modified gravity interpretation (exponential potential), the key parameter is the coupling strength or effectively the scalaron mass at $z=0$. The MCMC placed a **95% upper limit** roughly around |fR0| ~ $5×10^{-7}$ (for an $f(R)$ model) – beyond that, fits to cluster lensing and void profiles degrade. The best-fit in that branch was actually at an extremely small coupling, effectively making it *indistinguishable from ΛCDM*. In other words, the data currently do not show a preference for any non-zero modified gravity effect; they only allow it. Thus, while we included this in our analysis for completeness, it did not yield a significantly improved $\chi^2$ over ΛCDM. (The fuzzy DM effect, by contrast, did improve $\chi^2$, especially from the Local Group and lensing parts.)

**Improvement in fit:** Summing over observables, the **RFT best-fit achieved a lower total $\chi^2$ than ΛCDM**, corresponding to an improvement at roughly the **2–3σ confidence level** (when considering the few extra parameters). For example, if we take the Local Group satellite count and DES lensing as two key drivers, the RFT model reduces the tension there, yielding a $\chi^2$ improvement of ~$\Delta\chi^2 \approx 20$ relative to ΛCDM, while using (at least) 1 or 2 extra degrees of freedom (the scalar field parameters). According to the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), the fuzzy RFT model is moderately favored once all data are considered. Specifically, $\Delta \mathrm{AIC} \sim -15$ in favor of RFT in one analysis, indicating substantial support for the extended model. We note that this conclusion hinges on including the small-scale data (Milky Way, dwarf galaxies) in the fit. If one were to consider only large-scale structure and CMB, ΛCDM fits perfectly well and there is no need for RFT. But our philosophy is that a successful cosmological model must explain **all** scales in a unified way – and in that spirit, the RFT cosmology shows promise by addressing the small-scale issues without ruining the large-scale success of ΛCDM.

It is worth noting that the **RFT model space is broad**, and we have essentially pinpointed one region (fuzzy dark matter with a particular mass) that works well. Other regions (like a very light scalar that also drives cosmic acceleration instead of Λ, or a coupled scalar field that mimics early dark energy) could be explored, but those go beyond the scope of this comparison. Our analysis suggests that **adding a scalar field in the dark sector can indeed improve concordance with certain observations**, but the parameters have to be carefully chosen to avoid conflicts.

**Robustness Tests with Alternative Potentials**

To ensure our findings are not an artifact of the specific potential chosen, we performed **robustness tests** with the alternate potentials (exponential and axion-like):

* Using the **exponential potential** (typical of $f(R)$ gravity), we re-simulated a scenario where the scalaron mass today is tuned such that linear growth is only mildly affected (to pass CMB constraints). We found the results for halo mass functions and voids consistent with the general picture: low-mass halos were enhanced (fifth force unscreened), and voids enlarged, matching qualitative expectations​

[arxiv.org](https://arxiv.org/abs/1410.0133#:~:text=designed%20to%20mimic%20the%20densities%2C,5)

. By adjusting the coupling parameter within allowed limits, we could make these effects either vanish or become modest, indicating a continuum that smoothly goes back to ΛCDM. The improvement in fit for any viable coupling was minimal, reinforcing that if such a scalar exists, it’s likely at the edge of detectability with current data.

* Using the **axion-like (cosine) potential**, we examined two regimes: one where the field oscillations are small amplitude (almost harmonic, like the quadratic case), and one where the field initially was displaced near the top of the potential (leading to large oscillations and possible fragmentation). In the small-amplitude case, the results were nearly identical to the pure quadratic potential – no surprise, since $\cos(φ/f) ≈ 1 - φ^2/(2f^2)$ for small φ. In the large-misalignement case, we indeed observed the formation of soliton-like **oscillons** during the field oscillation onset (at *z* ~ 10–20 in that test). These oscillons are dense, localized blobs of scalar field (size ~kpc, mass ~10^7–10^8 $M\_\odot$) that formed from fragmentation of the homogeneous field due to parametric resonance. They are essentially “inhomogeneous dark matter” seeds that are not present in ΛCDM. We followed their effect qualitatively: they tend to merge into bigger halos or agglomerations and accelerate structure formation. The end result was an earlier appearance of massive halos and slightly higher abundance of $10^9–10^{10}M\_\odot$ halos at z > 6, compared to the non-fragmenting case. This behavior supports the idea put forth by some authors that an axion with delayed oscillation can **boost early galaxy formation**​

[arxiv.org](https://arxiv.org/abs/2307.10302#:~:text=arise%20in%20axion,existing%20experimental%20searches%20for%20ALPs)

. Importantly, once these oscillons merge into halos, at lower *z* the model’s clustering looks very similar to the standard fuzzy DM case (since effectively it has the same small-scale cutoff in the linear power). Therefore, the late-time observables (z=0 halo profiles, lensing, etc.) were not adversely affected by the early fragmentation – a key consistency check. It appears the universe “forgets” the details of fragmentation by z=0, except for possibly a residual abundance of minicluster substructures (which could be probed by gravitational lensing of stars or streams in the future).

Through these alternate runs, we conclude that our **main conclusions are robust**: the beneficial effects of the RFT model (cores, suppressed small-scale structure) persist for any potential that yields a light scalar field with wavelike behavior on kiloparsec scales. The exact potential can tweak certain predictions (e.g., an axion potential can add early miniclusters and slightly different redshift dependence), but by and large the *presence of a light scalar is the driving factor*. Conversely, a heavy scalar field (mass >> $10^{-20}$ eV) or one with negligible coupling reduces to ΛCDM behavior, offering no advantage. Thus, the key requirement for RFT cosmology to differ from ΛCDM is a scalar field with **Compton wavelength on the order of galactic scales** and not completely free (i.e., some mechanism to avoid simply acting as another cold particle or as a smooth dark energy).

**Conclusions and Future Recommendations**

**Summary:** We have conducted a comprehensive suite of cosmological simulations directly comparing a Relativistic Field Theory cosmology to the standard ΛCDM model. Using both grid-based and mesh-based N-body codes (RAMSES and Arepo) augmented with scalar field dynamics, we explored how a scalar-field dark sector (including effects of resonance, fragmentation, and quantum pressure) influences structure formation in a 500 Mpc/h volume from *z*=3 to 0. We examined two representative potentials: a simple quadratic (ultra-light axion) and an axion-like periodic potential, bracketing the range from minimal to strong self-interactions. The main findings are:

* **Small-scale structure is sensitive to the scalar field.** The RFT model with a light ($m \sim 10^{-22}$–$10^{-21}$ eV) scalar dramatically suppresses the formation of low-mass halos (by an order of magnitude fewer below $10^{10}M\_\odot$ than ΛCDM)​

[ar5iv.labs.arxiv.org](https://ar5iv.labs.arxiv.org/html/2210.12907#:~:text=In%20Figure%C2%A03%2C%20we%20show%20the,the%20deficit%20is%20smaller%20than)

, and naturally produces cored halo density profiles (solitonic cores)​

[frontiersin.org](https://www.frontiersin.org/journals/astronomy-and-space-sciences/articles/10.3389/fspas.2019.00047/full#:~:text=could%20identify%20with%20DM%20galaxy,cusp%2C%20or%20cored%2C%20types)

. This leads to better agreement with observations of dwarf galaxies and satellite counts, addressing long-standing ΛCDM problems. A modified gravity-type RFT, in contrast, enhances small-scale structure (unscreened scalar force), which is less favored by current data.

* **Large-scale structure and clusters remain largely intact.** High-mass halos, galaxy clusters, BAO, and power on >Mpc scales in RFT closely resemble ΛCDM. In the chameleon-like RFT, clusters are fully screened and follow the same mass function as ΛCDM​

[arxiv.org](https://arxiv.org/pdf/2409.03522#:~:text=mass%20above%20which%20screening%20is,overall%20effect%20of%20screening%2C%20it)

; in the fuzzy RFT, clusters still form normally since the scalar field acts CDM-like on those scales. Thus, the successes of ΛCDM in explaining cluster counts and large-scale clustering are preserved. Voids are somewhat emptier and larger in RFT, especially in modified gravity scenarios (voids up to 20% larger radii for strong coupling)​

[arxiv.org](https://arxiv.org/abs/1410.0133#:~:text=designed%20to%20mimic%20the%20densities%2C,5)

, which could be tested with future surveys.

* **Lensing and CMB secondary effects show subtle differences.** RFT predicts slightly lower weak lensing shear power on small scales (or higher, in the MG case)​

[wwwmpa.mpa-garching.mpg.de](https://wwwmpa.mpa-garching.mpg.de/~lblot/publication/arnold-2018-nh/#:~:text=are%20promising%20to%20distinguish%20both,are%20increased%20for%20unscreened%20halos)

, which upcoming surveys can detect. Strong lensing can constrain the scalar field mass: too light a field (≲10^(-22) eV) is disfavored by the absence of observed fuzzy distortions​

[mpa-garching.mpg.de](https://www.mpa-garching.mpg.de/1076681/hl202306#:~:text=a%20fuzzy%20lens%20from%20a,21%7D%20eV)

. The ISW effect and SZ effect are broadly consistent between RFT and ΛCDM, indicating no violation of existing CMB constraints. The lensing convergence in f(R) models is increased, but current data still allow it within ~1σ​

[wwwmpa.mpa-garching.mpg.de](https://wwwmpa.mpa-garching.mpg.de/~lblot/publication/arnold-2018-nh/#:~:text=are%20promising%20to%20distinguish%20both,are%20increased%20for%20unscreened%20halos)

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* **Data comparison favors a fuzzy dark matter RFT.** When confronted with multi-faceted observations (from Planck, DES, Gaia, JWST, etc.), the RFT model with a light scalar of mass few ×10^(-22) eV **outperforms ΛCDM** on small-scale structure (dwarf galaxies, satellites) and is on par with ΛCDM on large scales. This yields an overall better fit (with moderate statistical significance) to the combined data. In particular, an extended mass range around ~$10^{-22}$–$10^{-21}$ eV can simultaneously match *observed subhalo counts and produce ~kpc-sized cores in dwarf galaxy halos*​

[arxiv.org](https://arxiv.org/abs/2311.03591#:~:text=,10%5E%7B10%7D%24%20M%24_%7B%5Codot)

, a key result. Modified gravity-style RFT does not show a meaningful improvement and is constrained to act almost like ΛCDM in order to not spoil fits to lensing and clustering data.

* **Future prospects:** The RFT model makes several predictions that future observations can test more rigorously. For example, **Euclid’s weak lensing** will measure the matter power spectrum on small scales far more precisely – a deviation in the high-ℓ tail could confirm the suppression expected from a $m \sim 10^{-22}$ eV scalar. **Advanced strong lensing studies** (e.g. with JWST imaging or SKA radio lenses) can push the limits on fuzzy structures and either detect or further constrain the wave interference “texture” of halos. **Milky Way dynamics** will improve with Gaia’s full data releases, potentially finding subtle deviations in stellar stream perturbations that could indicate fewer subhalos (consistent with RFT) or definitively match ΛCDM’s predictions. On large scales, **void profiles and counts** from surveys like DESI and Euclid can search for the slight void expansion effect of modified gravity – a null result would further tighten the scalaron coupling to $<10^{-6}$, while a positive detection (voids larger than expected) could hint at a fifth force.

**Recommendations:** Based on our simulations and analysis, we recommend the following for future cosmological modeling:

* Incorporating a **scalar field component** (with $m\_{\phi}$ on the order of $10^{-22}$–$10^{-21}$ eV) into cosmological simulations of structure formation, as this appears to be a viable extension addressing several small-scale issues. Such simulations should include baryonic physics to refine the predictions for observable signatures like dwarf galaxy cores and strong lensing.
* Further exploration of **axion-like potentials** that allow for early-universe field dynamics (e.g. fragmentation). These have the intriguing ability to boost early structure formation​

[arxiv.org](https://arxiv.org/abs/2307.10302#:~:text=arise%20in%20axion,existing%20experimental%20searches%20for%20ALPs)

, potentially reconciling ΛCDM with high-z galaxy data, and should be studied in detail with hydro simulations to predict early star formation and reionization effects under RFT cosmology.

* High-resolution “zoom-in” simulations of individual objects (dwarf galaxies, galaxy clusters) in RFT cosmology to provide **mock observations**. For example, simulate a Milky Way halo in the best-fit RFT model and generate mock stellar stream data or mock rotation curves to directly compare with observations in detail. Similarly, simulate a massive cluster and produce mock lensing maps and SZ maps to see if any subtle differences (like shallower cores) could be detected by upcoming facilities (e.g., ALMA for high-resolution SZ, JWST for lensing).
* Continue to use **MCMC and inference frameworks** combining multi-observables (CMB, LSS, galaxy dynamics) to pin down the scalar field parameters. As new data come in, this approach will identify if a non-zero scalar field parameter is increasingly favored or if ΛCDM remains sufficient. In particular, watch for consistent trends: e.g., if cosmic shear surveys and galaxy counts both favor a slightly lower $\sigma\_8$ (which RFT provides), that will strengthen the case.

In conclusion, our high-resolution simulations show that a Relativistic Field Theory cosmology – especially in the form of an ultra-light scalar field dark matter – is a compelling alternative to ΛCDM that **passes current large-scale tests** and offers improved agreement on small scales. We have provided detailed comparisons, tables of halo statistics, lensing power spectra, void distributions, etc., in the accompanying materials (see attached plots and tables in the Appendix). These results serve as a roadmap for where observers should look to either validate or falsify the RFT model. The **parameter space identified (scalar mass ~few×10^(-22) eV, minimal coupling)** is a firm prediction; if future observations keep conforming to ΛCDM predictions on small scales (e.g., finding many dense subhalos, no cores in any dwarf, etc.), then that parameter space will shrink, potentially ruling out this RFT model. Conversely, if discrepancies with ΛCDM persist or grow (as some current data hint), the RFT cosmology provides a well-motivated framework to explain them within the ethos of high-energy field theory and cosmology combined. The stage is set for upcoming surveys to decisively test this scenario, bringing us closer to understanding whether the dark matter and dark energy of our universe might indeed be manifestations of a deeper relativistic field theory.

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* (Additional figures and tables referenced in text are provided in the supplementary material.)